A PARSIMONIOUS DESCRIPTION
OF THE HENDRY SYSTEM

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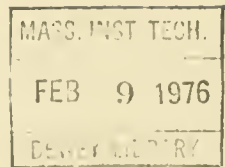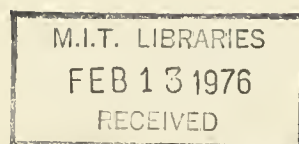WP 830-76   January 1976

ABSTRACT

In this paper we will present some basic concepts of the Hendry System and derive the results claimed in the HendroDynamics Chapters [2] from two simple probabilistic assumptions; namely, zero order consumers and switching proportional to market share. We will develop the notion of partitioning which constitutes a major component of the Hendry System by briefly describing the procedure for identifying the partitioning structure in a given market. With this illustration we will point out various implications which can be drawn from a correctly identified market structure. All of our results are obtained without resorting to the entropy concept that has caused so much confusion in recent debates over the validity or usefulness of the Hendry Model.

A PARSIMONIOUS DESCRIPTION OF THE HENDRY SYSTEM*

The Hendry Corporation has developed an innovative approach to consumer behavior -- HendroDynamics -- which has been receiving considerable attention recently. This paper focuses only on the description of some brand switching aspects of consumer behavior and the subsequent structuring of markets. The more involved HendroDynamics models which allegedly can be used to set optimal advertising and price levels are not discussed here. We do not have complete information on the Hendry Models and our understanding of HendroDynamics is based mostly on the material presented in the two privately printed chapters [2].

1. INTRODUCTION

While most of the results derived in this paper are presented in the two Hendry Chapters, the contribution of this paper lies in actually deriving those results from simple probabilistic assumptions. As we understand it, the Hendry Model is based on ideas which individually are simple, but in combination produce an interesting and useful "package." The first of these concepts is the "zero-order effect" assumption, which can be stated as follows:

Each consumer j has a probability p_{ij} of buying Brand i. On each purchase occasion, an individual consumer chooses among g brands on

---

the basis of a constant probability vector $(P_{1j}, P_{2j}, \ldots P_{gj})$. However, each consumer is not assumed to have the same purchase probability p_{ij} of buying Brand i. Thus, the model assumes a heterogeneous population of zero order consumers.[1]

In view of this assumption the Hendry Model can be classified as a heterogeneous multinomial probability model. For a product class with only two brands this model becomes a heterogeneous Bernoulli model. Thus, the Hendry Model uses the same starting point as some of the models developed in Morrison's dissertation which appear in Chapter 3 [4]. However, it is only fair to say that the Butlers have produced a richer set of properties, with more marketing insights than anything in [4].

For clearer understanding and convenience let us define various symbols and terms which will be introduced during the course of subsequent derivations. We have used the same notation found in [2] in order to facilitate comparisons with material printed by the Hendry Corporation.

N = Number of consumers in the product class.

N_i = Number of consumers buying Brand i on any particular purchase occasion.

g = Number of brands in the product class.

p_i = Market share of Brand i.

p_{ij} = Probability of consumer j choosing Brand i.

p_{ws} = Unconditional probability of switching from Brand s.

= Proportion of consumers switching from Brand s on any particular purchase occasion.

$P_{(s,s)}$ = Unconditional probability of repeat buying Brands s.

= Proportion of consumers repeat buying Brands s.

$P_{(s,h)}$ = Unconditional probability of switching from Brand s to Brand h.

$P_{h|s}$ = Probability of switching to Brand h for a consumer whose previous purchase is Brand s and who actually has switched out of Brand s. Note that this is a <u>double</u> conditional probability. It is conditional on having previously purchased Brand s <u>and</u> conditional on the subsequent purchase not being Brand s. For clarity it should be pointed out that this is not a Markovian transition probability, i.e. $P_{h|s} \neq \dfrac{P_{(s,h)}}{P_s}$. The Markovian transition probability $\dfrac{P_{(s,h)}}{P_s}$, is defined as the probability of purchasing brand h given previous purchase is Brand s. This Markovian probability, however, does not exclude the possibility of repeat purchasing Brand s. In terms of our previously defined quantities

$$P_{h|s} = \frac{P_{(s,h)}}{P_{ws}}$$

For clarity, it will be useful at this point to anticipate the section on partitioning. A partition is assumed to contain what the Hendry Model calls "directly competing brands." For a consumer who switches out of Brand i, his expected probability of buying Brand h is proportional to the market share of Brand h. That is,

$$P_{h|i} = k_i P_h \qquad \text{for all } i \neq h. \tag{1}$$

The above equation does <u>not</u> describe the behavior of an <u>individual</u> consumer switching out of Brand i. The probability, $P_{h|i}$, is the expected

conditional probability of buying brand h for a random consumer who switched out of Brand i. The Hendry Model <u>assumes</u> that this equation holds true within a partition and thus, equation (1) becomes the <u>definition</u> of a partition. This assumption has caused some confusion. Some researchers state unequivocably that the above equation does not hold true in a product class. However, the Hendry Model partitions a product class until equation (1) holds within each partition, but not necessarily for pairs of brands in different partitions. Again we repeat that (1) is essentially the Hendry <u>definition</u> for a partition. Equation (1) is not an empirical fact and it does not necessarily hold for all pairs of brands within an entire product class.

2. <u>LAW OF DETAILED BALANCING</u>

We will show here that this "law" discussed in [2] is a direct consequence of the zero-order assumption.

Consumer j's preference for Brand i is defined as the probability p_{ij} that consumer j chooses Brand i, such that

$$\sum_{i=1}^{g} p_{ij} = 1.$$

This equation implies that each consumer's purchase probabilities over all the g brands in the market add up to 1. Also, we have

$$\sum_{j=1}^{N} p_{ij} = E[N_i].$$

That is, summing the preferences for Brand i over all the N consumers gives the expected number of consumers who will buy Brand i. The actual

number of customers choosing Brand i, N_i, of course, is a random variable.
The preference vectors, p_{ij}'s, are assumed to be stationary over time.

Now let us consider two purchase decisions in which a consumer switches
brands. The probability of consumer j choosing Brand s the first time and
Brand h the second time is (by the zero order assumption)

$$P_{(s,h)_j} = p_{sj} \, p_{hj}.$$

where $P_{(s,h)_j}$ is the probability of consumer j switching from Brand s to
Brand h. The probable number of buyers switching from Brand s to Brand h
becomes

$$E[N_{(s,h)}] = \sum_{j=1}^{N} p_{sj} \, p_{hj}.$$

Similarly, the probable number of buyers switching from Brand h to Brand
s is

$$E[N_{(h,s)}] = \sum_{j=1}^{N} p_{hj} \, p_{sj},$$

from which

$$E[N_{(s,h)}] = E[N_{(h,s)}], \tag{2}$$

i.e., the expected number of buyers switching from Brand s to Brand h,
is equal to the expected number of buyers switching from Brand h to Brand s
at equilibrium   (i.e. when the probabilities in the preference vectors are
stationary -- constant -- over time.).

Since Brands h and s were chosen arbitrarily, the expected number of
buyers switching between each pair of brands is equal in both directions.
It follows from this that the expected total number of consumers switching
out of a brand is equal to the expected total number switching into the
brand. The zero-order process assumption, as shown above, is crucial in
the derivation of the balanced switching relationship. Also, the rela-
tionship holds good only under equilibrium conditions in the market since
we require brand preferences to be stationary.

3. SWITCHING CONSTANT K_w

The switching constant, K_w, plays a central role in describing
consumer behavior under a zero-order process. We will now derive its
value.

The probability of a consumer switching from Brand s to Brand h
may be expressed as

$$P_{(s,h)} = P_{ws} \, P_{h|s} \, . \tag{3}$$

The first term on the right, p_{ws}, is the probability of switching out
of Brand s. The second term, $p_{h|s}$, represents the probability of buying
Brand h for a consumer switching from Brand s. The term, $p_{h|s}$ can be
understood by noting that

$$\sum_{\substack{i=1 \\ i \neq s}}^{g} p_{i|s} = 1 \, . \tag{4}$$

That is, for a consumer switching out of Brand s his purchase probabilities sum up to 1 over the remaining brands. We know that the sum of market shares of all the g brands adds up to unity, that is

$$\sum_{i=1}^{g} p_i = 1.$$

Recall the Hendry definition of a particular stated earlier. Namely,

$$p_{h|s} = K_s \, p_h \tag{1}$$

That is, given that consumers switch from brand s, they switch to the remaining brands in proportion to the market shares of these other brands. From (4) substituting for $p_{i|s}$, we have:

$$K_s \sum_{\substack{i=1 \\ i \neq s}}^{N} p_i = 1.$$

Hence, the proportionality constant K_s is

$$K_s = \frac{1}{(1-p_s)} .$$

Note that although the concept embodied in the key assumption (1) is the same for all brands in the partition, the proportionality constant differs for each brand. From equation (1) using $K_s = \frac{1}{(1-p_s)}$, we have:

$$p_{h|s} = \frac{p_h}{1-p_s} . \tag{5}$$

Substituting for $p_{h|s}$ in equation (3) we find

$$P_{(s,h)} = p_{ws} \frac{p_h}{1-p_s} \quad .$$

Similarly,

$$P_{(h,s)} = p_{wh} \frac{p_s}{1-p_h} \quad .$$

From the Law of Detailed Balancing, we know that

$$P_{(s,h)} = P_{(h,s)} \; ,$$

and therefore

$$p_{ws} \frac{p_h}{1-p_s} = p_{wh} \frac{p_s}{1-p_h}$$

Dividing both sides of the equation above by $p_s p_h$, we have

$$\frac{p_{ws}}{p_s(1-p_s)} = \frac{p_{wh}}{p_h(1-p_h)} \quad . \tag{6}$$

Since Brands s and h were chosen arbitrarily, we can generalize equation (6) as

$$\frac{p_{wi}}{p_i(1-p_i)} = K_w \; , \tag{7}$$

for all brands i = 1, ..., g in the partition.

K_w in the equation above is termed the Switching Constant in the Hendry Model. The term, p_i, represents the market share of Brand i. If all the buyers of Brand i had identical probabilities of purchasing Brand i, p_i, then the denominator in equation (7), $p_i(1-p_i)$, would represent

the switching proportion from Brand i to the competitors of Brand i.
The numerator in equation (7), p_{wi}, represents the proportion of consumers
switching from Brand i under the heterogeneous multinomial probability
model assumption. The switching constant, K_w, is thus the ratio of
total brand switching in the actual heterogeneous multinomial probability
model to total brand switching that would result from a homogeneous multi-
nomial probability model that produced the same market shares. At equili-
brium, this ratio has the same numerical value for all brands, within a
partition. K_w ranges between 0 and 1 and the smaller its value the more
heterogeneous is the partition.

The switching constant, K_w, has several other implications in terms
of the switching behavior of consumers in the market which become clearer
upon algebraic manipulation of equations (3) through (7). Multiplying
equation (6) by $p_s \, p_h$, we have

$$p_s p_h \, \frac{p_{ws}}{p_s(1-p_s)} \;=\; p_s p_h \, \frac{p_{wh}}{p_h(1-p_h)} \; ,$$

from which we obtain, by substitution from equation (7),

$$\frac{p_{ws} \, p_h}{(1-p_s)} \;=\; p_s \, p_h \, K_w \; .$$

Finally, using equation (7), we find

$$p_{(s,h)} \;=\; K_w \, p_s \, p_h \; , \qquad\qquad (8)$$

where K_w is the switching constant.

Bass [1] has derived the brand switching relationship presented
in equation (8) using a different approach from the one presented here.

He obtains:

$$P_{(i,j)} = (1-\rho) \; p_i \; p_j \qquad\qquad (9)$$

where ρ represents the correlation between the purchases of any one of the g brands in a product class over two purchase occasions. Each product class is characterized by a single correlation measure, ρ, and it may be thought of as the "product class brand loyalty factor." In deriving this brand switching equation Bass [1] makes two assumptions: (i) each consumer is a zero-order process, and (ii) V_i's, the utilities for brands are distributed gamma over the population with the property that Var $[V_i]/E[V_i]$ = constant, for all brands i. The probability that a consumer chooses brand i is then his utility for brand i divided by the sum of the utilities for all brands, including brand i.

From equation (7), we have

$$P_{wi} = K_w \; p_i (1-p_i) \quad ,$$

from which the total switching for all brands becomes:

$$P_w = \sum_i P_{wi} = \sum_i K_w \; p_i (1-p_i) \; .$$

Solving for K_w, we obtain:

$$K_w = \frac{P_w}{\sum_i p_i \, (1-p_i)} \; . \qquad\qquad (10)$$

Thus, the switching constant K_w is applicable to the total product class as well as to the individual brands within the product class. In the Hendry Model, the switching constant as calculated from equation (7) for individual brands within a partition should equal the

switching constant obtained from equation (10) using total switching within the partition. Thus, within a partition there is one value of the switching constant which describes switching behavior for individual brands as well as the aggregate switching behavior within the partition.

The switching constant can be used in the calculation of repeat purchase and Markovian transition probabilities. The probability p_s of buying Brand s, equals the probability $p_{(s,s)}$ of having repeat purchased Brand s plus the sum of the probabilities $p_{(i,s)}$ of having switched from any other brand to Brand s. That is:

$$p_s = p_{(s,s)} + \sum_{\substack{i \\ i \neq s}} p_{(i,s)} .$$

Substituting for $p_{(i,s)}$ from equation (8) and with some simple algebraic manipulations, we find:

$$p_s = p_{(s,s)} + K_w p_s (1-p_s) .$$

From which, we have:

$$p_{(s,s)} = (1-K_w) p_s + K_w p_s^2 . \tag{11}$$

The Markovian repeat transition probabilities are given by:

$$\frac{p_{(s,s)}}{p_s} = (1-K_w) + K_w p_s , \tag{11a}$$

which increases as p_s increases.

Using equation (8)

$$\frac{p_{(s,h)}}{p_s} = K_w p_h , \tag{11b}$$

which is <u>independent</u> of s.

As an illustration, it would be useful to examine various consumer statistics for a typical value of K_w. Consider a three-brand product class with brand shares 0.5, 0.3 and 0.2. We have chosen a value of 0.4 for the switching constant. This value of K_w is arbitrary but it is approximately the value which would be obtained as a theoretical estimate in the Hendry System for this product class using notions of entropy [2]. Table 1 contains "Markovian" repeat purchase and switching probabilities for this three-brand case which are obtained using equations (11a) and (11b).

TABLE I

Buyers of Brand #	PROPORTION WHO NEXT BUY		
	1	2	3
1	.8	.12	.08
2	.2	.72	.08
3	.2	.12	.68
All Buyers	.5	.3	.2

Total Switching = 24.8% *

We find in Table 1 that switching probabilities (conditional) from any brand (say, #2, or #3) to a particular brand (#1 in this case) to be identical (because of equation (11b)). The repeat purchase probability (conditional) is found to be higher for larger brands; this result is in general agreement with empirically observed data.


* Total switching = $p_w = K_w \sum_i p_i (1-p_i) = .248$

4. <u>PARTITIONING</u>

In this section, we shall discuss the notion of partitioning -- the
hierarchy of market structure relationships. From a marketing practitioner's
view-point this is operationally one of the most important concepts under-
lying the Hendry approach. The Hendry Model posits that consumer prefer-
ences partition markets into a hierarchy of product set structures. For
instance, the coffee market may be partitioned into instant vs. ground
coffee and then within each category into caffeinated vs. decaffeinated
brands. Consumer alternatives are substitutible <u>within</u> the smallest set
to which they belong. (e.g. all brands within the set of instant-decaf-
feinated coffee).

<u>A Hypothetical Illustration of Market Structure for the Coffee Market</u>

```
                            Coffee
                              |
          _____
         |                                           |
      Instant                                      Ground
         |                                           |
    _____                              _____
   |             |                            |             |
Caffeinated  Decaffeinated              Caffeinated  Decaffeinated
```
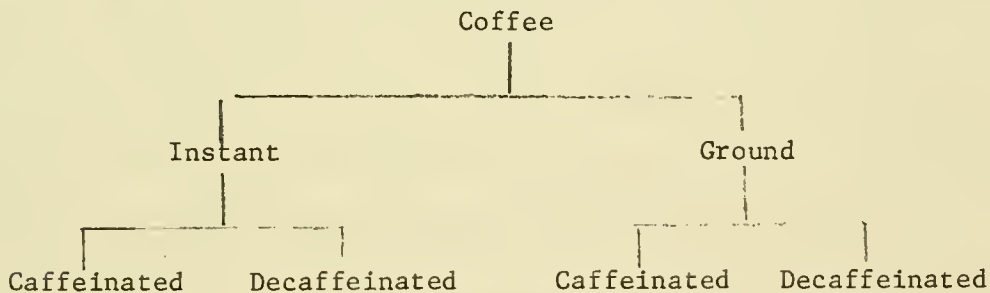
A correctly identified preference structure provides the appropriate
competitive frame for assessing the relative performance of brands in terms
of market share, sales volume, profitability, etc.. The preference struc-
ture gives an idea of the relative influence of marketing strategies and
thus, helps in the development of strategic marketing plans. In the
Hendry System, the structure of partitioning within a product class is
determined by analyzing actual consumer brand switching behavior. This

is in opposition to the general practice of discovering the structure of partitioning through the judgment of marketing managers and/or consumers' perceptions of the brands. The relative merits of these different approaches are not discussed in this paper.

Following the Hendry approach, a hypothetical partitioning structure based on "expert judgment" is set up and switching patterns are examined given this structure. With some experience in identifying "true" partitioning structures in markets, a certain amount of knowledge or learning is acquired from every partitioning hypothesis that is tested for the market under consideration. Upon one or more of these iterative attempts a market structure is identified which "fits" the empirical data reasonably well.

For instance, in the margarine market, the form of margarine (stick, cup, etc.) might represent the primary level of decision making. That is, a consumer decides first on the form of margarine she wishes to buy, and only then chooses a particular brand within that form. The structure of the market upon analysis may look as follows:

A Form-Primary Market

MARGARINE MARKET

CUPS   STICKS

B_{c3} B_{c2} B_{c1} B_{s2} B_{s1}
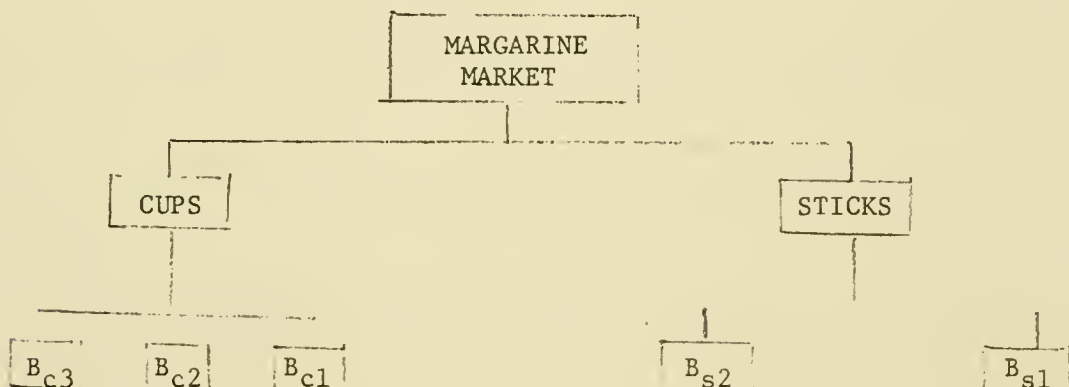
As shown in the figure above, in this form-primary margarine market we assume there are two forms of margarine (cups and sticks) and three brands (1, 2 and 3). Brand labels 1 and 2 are sold in both forms whereas Brand 3 margarine is sold only in cups. In order to describe some of the switching patterns which will operate in a form-primary margarine market, let us define some terms:

P_{cups} = Proportion of margarine sold in cup form.

P_{sticks} = Proportion of margarine sold in stick form.

P_{ci} = Market share of Brand B_{ci} within "cups" partition, $i = 1, 2, 3$.

P_{si} = Market share of Brand B_{si} within "sticks" partition, $i = 1, 2$.

K_{wf} = Switching constant describing switching <u>across</u> forms of margarine .

K_{wc} = Switching constant <u>within</u> the partition of cups form.

K_{ws} = Switching constant <u>within</u> the partition of stick form.

Following the derivations in earlier sections, the expected proportion of buyers switching between forms of margarine is given by:

$$\text{Total switching across forms} = K_{wf} \, P_{cups} \, P_{sticks} \qquad (12a)$$

Note that the Hendry Model allows a consumer to purchase a set of Brands that are not all necessarily in the same partition. The switching level from a Brand B_{ci} to a stick form of margarine would be given by:

$$\text{Switching from } B_{ci} \text{ to sticks} = p_{ci}(K_{wf} \, P_{cups} \, P_{sticks}) \, . \qquad (12b)$$

Similarly,

$$\text{Switching from } B_{si} \text{ to cups} = p_{si} \, (K_{wf} \, P_{cups} \, P_{sticks}) \, . \qquad (12c)$$

For switching patterns within a partition we have:

$$\text{Switching from } B_{ci} \text{ to } B_{cj} = K_{wc} \, P_{ci} \, P_{cj} \, . \qquad (12d)$$

Likewise

$$\text{Switching from } B_{si} \text{ to } B_{sj} = K_{ws} \, P_{si} \, P_{sj} \, . \qquad (12e)$$

The switching constant for each brand within a partition should be identical and it should be equal to the switching constant describing aggregate switching behavior in that partition.

Loosely stated the Hendry partitioning algorithm would try various partitioning schemes until one was found where all of the numerous relations (12) are "satisfied." Obviously, some apriori notions of reasonable partitions are needed since the total number of possible partitions in any real market would be astronomically large. Admittedly, there is a certain ad hoc nature to this approach, but there is a clear cut, well defined principle being utilized. Namely, each partition consists of a set of heterogeneous multinomial consumers in which each brand in the partition has the same switching constant and this switching constant also applies to the partition as a whole.

Thus, in the hypothetical illustration above, each form of margarine would represent a separate partition with its own switching characteristics. The margarine market can, in such a case, be classified as form primary. A consumer in such a form primary market will have a very small preference for a brand which does not carry her margarine form. That is, the form of margarine would be the primary structural component because the consumer would first identify margarine as having a particular form before choosing a brand within that form.

The identification of the partitioning structure in a market has several practical implications. These implications can be best described

by comparing the above structure with a brand-primary margarine market.
Using the assumptions above, a brand-primary margarine market can be
represented in a figure as shown below.

A Brand Primary Market
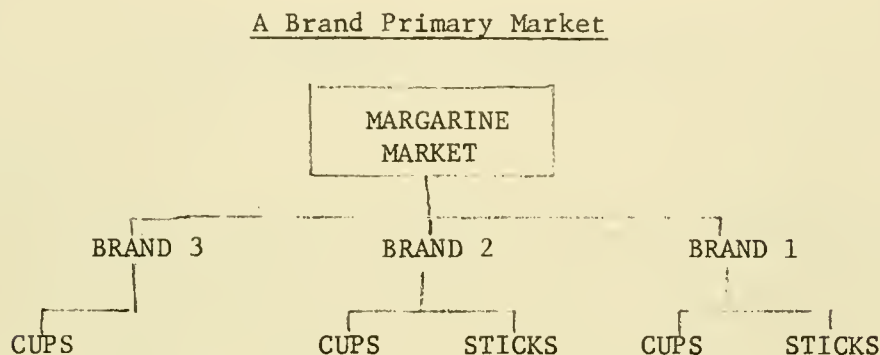
Firstly, in the form-primary market, the consumers exhibit lower
brand-name loyalties than in the brand-primary market. Secondly, five
separate brands are perceived by consumers in the form-primary margarine
market. The firm manufacturing Brand 1 (say, Firm 1) has to promote its
two forms of margarine separately. On the other hand, in the brand-primary
case, Firm 1 could promote its two product types together. Finally,
if Firm 3 wanted to introduce margarine in stick form in the brand primary
case it would experience a certain amount of "cannibalism." On the other
hand, in the form-primary case, if Firm 3 introduced margarine in stick
form, it would end up getting a share in the "sticks" partition which
would be independent of its share in the "cups" partition. These conclu-
sions are of course overstated , but the general tone of these implications
would be helpful in practice.

In concluding this section on partitioning, it should be pointed out
that the Hendry Model identifies a partitioning structure which is con-

sistent with numerous relations (e.g. equations (12) defined by the model.)
Whether this structure is the _true_ partitioning structure would of course
need further verification. The Hendry partitioning might be consistent over
time (i.e. reliable), but the validity of this structure would still require
other supporting data.

5. MODEL ANALYSIS AND ESTIMATION OF K_w

Model Analysis:

A heterogeneous multinomial probability model is completely determined
by the knowledge of preference distributions for individual brands in the
product class. Various consumer statistics like repeat purchase rates,
switching rates and the switching constant can be easily deduced from knowledge
of the preference distribution. In this section, we shall estimate K_w from
a knowledge of the preference distribution for two different cases of heter-
ogeneity in the consumer population.

Arbitrary Heterogeneity:

Let us suppose we are interested in analyzing a particular brand.
Then, at each trial, or purchase occasion, the consumer can purchase either
the brand under consideration (Brand 1) or some other brand in the aggre-
gate "all other" class (Brand 0). Under the "zero-order process" assumption,
on every trial the process has a probability p of being in State 1 (purchase
of Brand 1) regardless of the past history of the process. The purchase pro-
bability p may have an arbitrary distribution f(p) over individuals in
the population. Let μ_1 and μ_2 denote the first and second moments of this
distribution about origin. We have:

$$\mu_1 = \int_0^1 p \, f(p) \, dp \, .$$

The first moment, μ_1, above represents the market share of Brand 1, p_1. The second moment about the origin is a measure of the spread of the preference distribution of Brand 1 and is given by:

$$\mu_2 = \int_0^1 p^2 \, f(p) \, dp.$$

The quantity μ_2 in the equation above represents the second raw moment and not the variance of the preference distribution $f(p)$. We assume these values of the first two moments about the origin are known to us. From equation (18) we have for Brand 1

$$K_w = \frac{P_{w1}}{p_1(1-p_1)} \, .$$

The numerator, P_{w1}, in the equation above represents the level of switching between Brand 1 and all other Brands in the product class. We have

$$P_{w1} = \int_0^1 p(1-p) \, f(p) \, dp = (\mu_1 - \mu_2).$$

The switching constant, K_w, can therefore be obtained from the knowledge of the first and second moments of the preference distribution $f(p)$. This yields

$$K_w = \frac{(\mu_1 - \mu_2)}{\mu_1(1- \mu_1)} \, . \tag{13}$$

Beta Heterogeneity:

Once again, we assume that each consumer has a probability p of buying Brand 1 and the complementary probability (1-p) of buying Brand 0 (usually an aggregated "all other" brand). However, the purchase probability p is assumed to have a specific distribution; namely, beta. The beta distribution

has the following form:

$$b(p) = \frac{\Gamma(m+n)}{\Gamma(m) \cdot \Gamma(n)} \quad p^{m-1} (1-p)^{n-1}, \quad \text{for } 0 < p < 1$$

$$= 0, \text{ otherwise.}$$

where $\Gamma(\cdot)$ is the gamma function and the parameters m, n are > 0. The first
and second moments (μ_1, μ_2) about the origin, for this distribution, are:

$$\mu_1 = \frac{m}{m+n} \quad ,$$

$$\mu_2 = \frac{m(m+1)}{(m+n)\ (m+n+1)} \quad .$$

It should be noted that selection of a beta distribution for analysis here
is based on its flexibility, tractability and successful applicability in
various stochastic buyer behavior models. We can get a feel for the flex-
ibility of the Beta distribution by noting the various shapes it can take.
When m and n are less than one, it is "U shaped"; the bulk of population
is concentrated near the extremes, p = 0 and p = 1. For m and n less
than 1, it takes a J-shape or a reversed J-shape accordingly as m is greater
than or less than n. When m and n both exceed 1, the beta distribution is
bell-shaped, and skewed left or right accordingly as m is greater or less
than n. As m and n grow larger while m $\simeq$ n, the beta distribution approaches
the familiar normal distribution. Finally, when m = n = 1, the beta dis-
tribution becomes a uniform distribution.

The switching constant, K_w, can be estimated for this case in two ways.
Firstly, K_w can be estimated by substituting for the first and second
moments. We have:

$$K_w = \frac{(\mu_1 - \mu_2)}{\mu_1(1-\mu_1)} \quad .$$

Substituting the expressions for the two moments yields

$$K_w = \frac{m+n}{m+n+1} \quad .$$

A second way of estimating K_w is by deducing the value of switching level P_{w1} for Brand 1 given beta heterogeneity.[2]

$$p_{w1} = P(10) = P(0|1) \; P(1) = \left(\frac{n}{m+n+1}\right)\left(\frac{m}{m+n}\right).$$

Using this expression for the numerator of the switching constant we find once again that

$$K_w = \frac{m+n}{m+n+1} \quad . \tag{14}$$

In deriving the value of the switching constant and other results presented in Section 3 we made two assumptions -- namely, the zero order assumption and that switching is proportional to market share (equation 5). It is useful to note here that the relationship of switching proportional to market share

$$p_{h|s} = \frac{p_h}{1-p_s} \tag{5}$$

holds true for a dirichlet distribution which is a multivariate analogue of the beta distribution.

Estimation of K_w:

Empirically, it is possible to estimate K_w from equation (10). We find that estimation of K_w requires knowledge of market shares of various brands

and the total switching in the product class. This information can be
obtained without much difficulty from consumer panel data.[3] Thus, this
method of estimating K_w does not require the knowledge of the distribution
of probabilities across the population of consumers.

If we assume a functional form for the distribution of probabilities,
we could estimate the parameters of this distribution. Then, the switching
constant will be a function of these estimated parameters (e.g., equation (14)).

An Algorithm for Applying Hendry Approach

The Hendry approach in so far as it deals with description of the brand
switching aspects of consumer behavior and structuring of markets can be
employed in a practical situation by following the sequence of steps given
below.

1. Set up a hypothesis about the nature of partitioning in the markets.
    Such a hypothesis is typically developed on the basis of managerial
    judgment.

2. Analyze the actual switching behavior by calculating empirically
    the values of the switching constant both across and within par-
    titions given the hypothesised partitioning structure of Step 1.
    If the switching patterns within and across partitions are found
    to be consistent with the partitioning hypothesis, as illustrated
    in the section on partitioning, the "true" Hendry partitioning struc-
    ture has been identified. (As we can recall from the section
    on partitioning, within each partition every individual
    brand should have approximately the same value for switching
    constant.) If, however, the switching patterns are found to be
    inconsistent, return to Step 1 and set up a new hypothesized
    partitioning structure.

6. CONCLUSION

In this paper we have presented what we believe to be the basic framework of the Hendry Model. As described in this paper the Hendry approach leads to several interesting conclusions about consumer markets.

Following the Hendry approach, a consumer market can be divided into a set of mutually exclusive and exhaustive partitions. Each of these partitions is treated as a distinct market and can be modeled according to a zero-order process. That is, within each partition consumer behavior is described by a heterogeneous multinomial probability model. The level of switching within a heterogeneous multinomial probability model is less than that for a homogeneous multinomial probability model. Hence, the value of the switching constant, within a partition, is less than unity. Within a partition, each brand is required to have the same value of the switching constant. As this value of K_w decreases it implies a higher degree of heterogeneity in consumer brand preferences.

From a practical viewpoint partitioning is one of the most significant concepts of the Hendry System. A knowledge of partitioning is a given market can be instrumental in the development of a sound promotional strategy. Further, it provides valuable information for new brand introduction in the given market -- particularly with respect to cannibalization.

The brand switching behavior results derived in this paper are based on two assumptions -- the zero order assumption and switching proportional to market share. Chapter 3 of Massy et. al. [4] provides three different procedures to test this zero order assumption.

The testing of the second assumption of switching proportional to market share requires that we have a prior description of the market structure relationships (e.g., from managerial judgment, or from consumers' perceptions of the brands). We can then test if the second assumption holds true within a partition. It would be meaningless to test the switching proportional to market share assumption in the Hendry Model framework since this relationship is the very definition used to identify a partition.

As shown in the procedural algorithm of the last section, application of the Hendry approach does not require the use of maximum entropy criterion. HendroDynamics does have a procedure to obtain a theoretial estimate of K_w which is based on the use of maximum entropy criterion. According to the Butlers, it is necessary to compute the theoretical value of the switching constant in consumer markets which are not in equilibrium. Also, the entropy concept is used as a system property of a consumer market for the advertising model of the Hendry System. However, we have avoided any discussion of entropy in this paper for two reasons: i) this concept of entropy in the Hendry System has caused a great deal of needless controversy, and ii) entropy is not needed to derive anything we have done in this paper. Chapters 5 and 6 of Kalwani's dissertation [3] go into considerable detail on the topic of entropy in the context of brand switching models.

One final comment is in order. We believe that the Hendry (and other similar) approaches are nothing more than parsimonious descriptions of consumer brand switching behavior; albeit, very useful and insightful descriptions. Others argue strongly that these approaches constitute a theory of switching behavior. We have only focused on the descriptive aspects in this paper, electing to leave the theorizing to some of our stochastic model building colleagues.

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FOOTNOTES

1. We should emphasize here that the zero-order effect assumption implies:
    (i) Any possible learning effects upon reaching equilibrium are of no
    consequence subsequently. (ii) Virtually all the brand switching data
    come from consumer panels where the unit is the household and not a
    particular individual. Hence, our zero-order model does not provide for
    different vectors for a household if purchasing is done by more than one
    family member. (iii) A zero-order model implies that consumers have
    stable attitudes towards various brands and thus constant purchase
    probabilities. It does not consider consumers to be in an exploratory
    or search process prior to their forming stable brand preferences.

    In actual practice, however, it can be argued that the zero-order effect
    assumption does not imply these severe restrictions but only that these
    three effects "wash-out" on aggregation of buyer behavior.

2. See Massy et. al. [3], Chapter 3, for a complete discussion of $p(0|1)$ and
    similar conditional probabilities for heterogeneous Bernoulli models.
    Essentially an unconditional beta prior is being updated by a binomial
    sample outcome to a conditional beta posterior distribution. In the
    specific case here for $p(0|1)$ the mean of this conditional posterior
    distribution $n/(m+n+1)$.

3. Consumer Panel data is based on the purchase diaries of a representative
    panel of a large number of households scattered throughout the country.
    Each household reports in its diary the date the product was purchased,
    brand chosen, store visited, retail price, package size, number of units
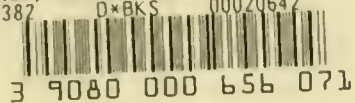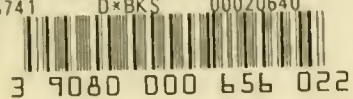   bought and if it was a deal purchase.